TD 2,5 – Déterminants

1. Soient $a, b, c \in \mathbf{C}$. Calculer les déterminants suivants :

$$d_1 = \begin{vmatrix} 7 & 11 \\ -8 & 4 \end{vmatrix}; \quad d_2 = \begin{vmatrix} 1 & 0 & 6 \\ 3 & 4 & 15 \\ 5 & 6 & 21 \end{vmatrix}; \quad d_3 = \begin{vmatrix} 1 & 2 & 0 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{vmatrix}; \quad d_4 = \begin{vmatrix} 1 & 0 & -1 \\ 2 & 3 & 5 \\ 4 & 1 & 3 \end{vmatrix};$$

$$d_5 = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix}; \quad d_6 = \begin{vmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}; \quad d_7 = \begin{vmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 1 & 3 \\ 2 & 1 & 0 & 6 \\ 1 & 1 & 1 & 7 \end{vmatrix}$$

$$d_8 = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}; \quad d_9 = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 2 & 3 & 1 & 1 \end{vmatrix}; \quad d_{10} = \begin{vmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{vmatrix};$$

$$d_{11} = \begin{vmatrix} 10 & 5 & -5 & 15 \\ -2 & 7 & 3 & 0 \\ 8 & 14 & 0 & 2 \\ 0 & -21 & 1 & -1 \end{vmatrix}; \quad d_{12} = \begin{vmatrix} a & a & b & 0 \\ a & a & 0 & b \\ c & 0 & a & a \\ 0 & c & a & a \end{vmatrix}; \quad d_{13} = \begin{vmatrix} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 \\ a & 0 & a & 0 & 3 \\ b & a & 0 & a & 0 \\ 0 & b & 0 & 0 & a \end{vmatrix}$$

2. Soit $n \in \mathbf{N}$. Soient $a, b, a_1, \dots, a_n \in \mathbf{C}$. Calculer les déterminants suivants, de taille n :

$$d_{14} = \begin{vmatrix} a_1 & a_2 & \cdots & a_n \\ a_1 & a_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & a_2 \\ a_1 & \cdots & a_1 & a_1 \end{vmatrix}; \qquad d_{15} = \begin{vmatrix} 1 & & & 1 \\ 1 & \ddots & (0) & \\ & \ddots & \ddots & \\ (0) & & 1 & 1 \end{vmatrix};$$

$$d_{16} = \begin{vmatrix} a+b & a & \cdots & a \\ a & a+b & \ddots & \vdots \\ \vdots & \ddots & \ddots & a \\ a & \cdots & a & a+b \end{vmatrix}.$$

Solutions. $d_1 = 116$; $\quad d_2 = -18$; $\quad d_3 = 6$; $\quad d_4 = 14$;
$d_5 = 96$; $\quad d_6 = -1$; $\quad d_7 = -12$;
$d_8 = a^3 + b^3 + c^3 - 3abc$; $\quad d_9 = -1$; $\quad d_{10} = -16$;
$d_{11} = 7\,700$; $\quad d_{12} = b^2c^2 - 4a^2bc$; $\quad d_{13} = 4a^3 + 27b^2$.
$d_{14} = (-1)^{n+1} a_1 (a_2 - a_1)^{n-1}$; $\quad d_{15} = 1 + (-1)^{n+1}$;
$d_{16} = (na + b) b^{n-1}$.

TSI2 — TD 2,5 – Déterminants — 2021-22

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2. Soit $n \in \mathbf{N}$. Soient $a, b, a_1, \dots, a_n \in \mathbf{C}$. Calculer les déterminants suivants, de taille n :

$$d_{14} = \begin{vmatrix} a_1 & a_2 & \cdots & a_n \\ a_1 & a_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & a_2 \\ a_1 & \cdots & a_1 & a_1 \end{vmatrix}; \qquad d_{15} = \begin{vmatrix} 1 & & & 1 \\ 1 & \ddots & (0) & \\ & \ddots & \ddots & \\ (0) & & 1 & 1 \end{vmatrix};$$

$$d_{16} = \begin{vmatrix} a+b & a & \cdots & a \\ a & a+b & \ddots & \vdots \\ \vdots & \ddots & \ddots & a \\ a & \cdots & a & a+b \end{vmatrix}.$$

Solutions. $d_1 = 116$; $\quad d_2 = -18$; $\quad d_3 = 6$; $\quad d_4 = 14$;
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